HOMEWORK 3 - V0

CSC2547/STA4273 WINTER 2019

University of Toronto

VERSION HISTORY:

1. Rademacher Complexity and Massart's lemma.

1.1. *Symmetrization for concentration [10pts].* For Q2.3 in Assignment 2, obtain a similar concentration result using a symmetrization argument. The steps will be as follows.

1. Use McDiarmid's inequality on the empirical process.
2. Find the Rademacher complexity of $\mathcal{F}$, linear functions constrained in ℓ_2-ball.
3. Use Talagrands contraction to bound $\mathcal{R}_n(f \circ \mathcal{F})$.

1.2. *Rademacher complexity of linear functions constrained in ℓ-1 ball [10pts].* For $i = \{1.2, ..., n\}$. we assume that data points z_i, have bounded coordinates, i.e., $\|z_i\|_\infty = \max_j |z_{ij}| \leq \kappa$ almost surely. Find an upper bound on the Rademacher complexity of linear functions constrained in a ℓ_1 ball of radius r, i.e.,

$$\mathcal{F} = \{f \ : \ f(z) = \langle \beta, z \rangle, \quad \|\beta\|_1 \leq r\}. \tag{1.1}$$

Compare your result to the Rademacher complexity of the ℓ_2 ball.

Hint: Region defined by the ℓ_1 ball has corners at re_j where e_j is the j-th standard basis vector. Therefore, $\{\beta \ : \ \|\beta\|_1 \leq r\} = \text{convex-hull}(\cup_j \{\pm re_j\})$. First, find the Rademacher complexity of the union (which is finite so Massart's lemma can help), then argue that this is equal to the complexity of the convex hull.

1.3. *Generalization of binary classification [10pts].* In a binary classification problem, we have a dataset of n iid (x_i, y_i) feature-response pairs where $x_i \in \mathbb{R}^d$ with $\|x_i\|_\infty \leq \kappa$ and $y_i \in \{-1, +1\}$. Learning task involves fitting a function of the form $f_\beta(x) = \text{sign}(\langle \beta, x \rangle)$ where $\|\beta\|_1 \leq r$. We noticed that 0-1 loss function is not smooth; therefore, we use capped-hinge loss as a relaxation which is given as $l((y, x), \beta) = \min\{2, \max\{0, 1 - y\langle \beta, x \rangle\}\}$.

Derive a generalization bound for the empirical risk minimizer.

1.4. *Course evaluation [5pts].* Did you complete the course evaluation for this course? Your answer should be yes or no, and there is only one correct answer. Hint: I would really appreciate your feedback on the course.