Tutorial 1

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Vector Institute
Machine Learning Black & Indigenous Program - Fall 2022

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Tutorial Outline[1]

1. Course Tips
2. ML Concepts Review
3. Nearest Neighbors Algorithm Review
4. Probability Review

[1] Slides adapted from Erdogdu and Zemel

Course Tips

As with any rewarding course, this one can be challenging and time-consuming!

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- **Assignments:** Start early! Try to determine what parts of lecture + tutorial are relevant. Attend office hours.
- **Office Hours and D2L Discussions Forum:** Ask questions you didn't get to during lecture or tutorial.
- **Others:** Attend talks by guest speakers, make use of Vector resources, network, etc.

The Machine Learning Problem

Building blocks for a machine learning problem:

- Stages
- Data
- Hyperparameters

Stages

- **Learning:** Extract information from data to make predictions.
- **Evaluation:** Check how well the algorithm/model makes predictions.

Data

For supervised learning problems, we often have data of the form:
n input data samples, each with d features:

$$\begin{bmatrix} x_{11} & x_{12} & x_{13} & \dots & x_{1n} \\ x_{21} & x_{22} & x_{23} & \dots & x_{2n} \\ \hdotsfor{5} \\ \hdotsfor{5} \\ x_{d1} & x_{d2} & x_{d3} & \dots & x_{dn} \end{bmatrix}$$

n targets, corresponding to each input sample:

$$\begin{bmatrix} y_1 & y_2 & y_3 & \dots & y_n \end{bmatrix}$$

Data

- **Training:** Used at learning stage to extract information from data relevant to predictive task and potentially transfer knowledge from data to make predictions later.
- **Validation:** Used to select possible algorithms and tune hyperparameters by mimicking test time behavior. Serves as a proxy to measure overfitting. (Hidden during learning.)
- **Test:** Used to evaluate model performance. (Unavailable to learner before evaluation stage.)

Hyperparameters

We would like to design an algorithm or learn parameters of the model based on the training data.
However, there are some (hyper)parameters that we, the designers of the algorithm, must determine.

- Knobs that we tune to find a right setting for the algorithm.
- Use validation data to choose the setting of a knob.

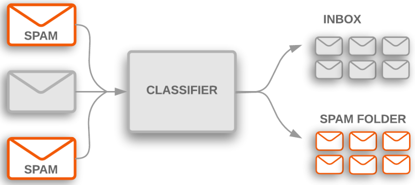
Hyperparameters

Examples:

- **Learning Rate:** size of updates made to parameters
- **Batch Size:** amount of the data used at every step of learning
- **k:** number of Nearest Neighbors

Classification

Given some data, we want to assign it to meaningful categories by learning the patterns in the training data.
How do we store information about the learned patterns?



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[2] developers.google.com/machine-learning/guides/text-classification/

Classification[3]

- **Option 1:** We don't! Like the NN algorithm, we can just look at the entire training data.
  Non-Parametric classifier.
- **Option 2:** We create a model with some parameters. During the learning stage, we store information in these parameters. During evaluation, we look at the learned model only.
  Parametric classifier.

[3] `https://machinelearningmastery.com/parametric-and-nonparametric-machine-learning-algorithms/`

Nearest Neighbors

Let's review the stages in the NN algorithm:

- **Learning:** None! This algorithm holds all the relevant information in the training set.
- **Evaluation:** For every test point, find the training point "close" to it and assign it the same category.

This needs us to define a notion of "closeness".

Nearest Neighbors

- "Closeness" is measured as a distance between the input vectors.

  For instance, the Euclidean norm: $\left\| \begin{matrix} x_{11} & - & x_{12} \\ x_{21} & - & x_{22} \\ \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots \\ x_{d1} & - & x_{d2} \end{matrix} \right\|_2$

- The NN algorithm compares these distances to determine the closest neighbor.

Probability Review

Let's review some probability basics that we will use in future lectures.

Why do we care about probability?

Uncertainty arises through:

- Noisy measurements
- Variability between samples
- Finite size of data sets

Probability provides a consistent framework for the quantification and manipulation of uncertainty.

Sample Space

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Events $E \subset \Omega$ are subsets of the sample space.

Sample Space

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Event is a subset of the sample space (eg. the event where both flips have the same outcome)
$E = \{HH, TT\} \subset \Omega$

Probability

The probability of an event E, $P(E)$, satisfies three axioms:

1: $P(E) \geq 0$ for every E

2: $P(\Omega) = 1$

3: If $E_1, E_2, \ldots$ are disjoint (cannot occur at the same time) then the probability of the union of all the events is the summation of their individual probabilities

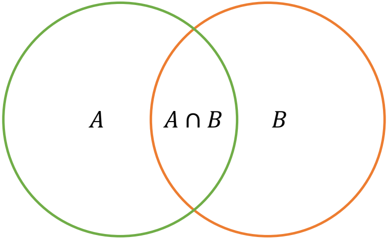
$$P(\bigcup_{i=1}^{\infty} E_i) = \sum_{i=1}^{\infty} P(E_i)$$

Joint and Conditional Probabilities

Joint Probability of A and B is denoted $P(A, B)$.

Conditional Probability of A given B is denoted $P(A|B)$.

Joint: $p(A, B) = p(A \cap B)$



Conditional: $p(A|B) = \frac{p(A \cap B)}{p(B)}$

$$p(A, B) = p(A|B)p(B) = p(B|A)p(A)$$

Conditional Example

Probability of passing the midterm is 60% and probability of passing both the final and the midterm is 45%.
What is the probability of passing the final given the student passed the midterm?

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What is the probability of passing the final given the student passed the midterm?
(Solution:

$$\begin{aligned} P(F|M) &= P(M, F)/P(M) \\ &= 0.45/0.60 \\ &= 0.75 \end{aligned}$$

)

Independence

Independence: Occurrence of A does not affect the occurrence of B

Events A and B are **independent** if $P(A, B) = P(A)P(B)$.

Independence

Suppose you have 2 coins. Coin 1 always comes up Heads and Coin 2 always comes up Tails. You close your eyes, pick a coin and toss it. Then you replace it, pick again and toss again.

- **Independent:** Before seeing the result of any toss, you wonder about 2 events; A: first toss is Head, B: second toss is Head.

$$P(A, B) = P(A)P(B) = 0.5 \times 0.5$$

- **Not Independent:** Now you wonder about the same events A and B but you toss the same coin twice.

$$\begin{aligned} P(A, B) &= P(B|A)P(A) \\ &= 1.0 \times 0.5 \\ &= 0.5 \neq P(A)P(B) \end{aligned}$$

Conditional Independence

Events A and B are conditionally independent given C if

$$P(A, B|C) = P(B|C)P(A|C)$$

Consider two coins[4]: A regular coin and a coin which always outputs heads.

$$\begin{aligned} A &= \text{The first toss is heads;} \\ B &= \text{The second toss is heads;} \\ C &= \text{The regular coin is used} \\ D &= \text{The biased coin is used} \end{aligned}$$

Then A and B are conditionally independent given C and given D.

[4] www.probabilitycourse.com/chapter1/1_4_4_conditional_independence.php

Conditional Dependence

Events A and B are **conditionally independent** given C if

$$P(A, B|C) = P(A|C)P(B|C)$$

Consider a coin which outputs heads if the first toss was heads, and tails otherwise.

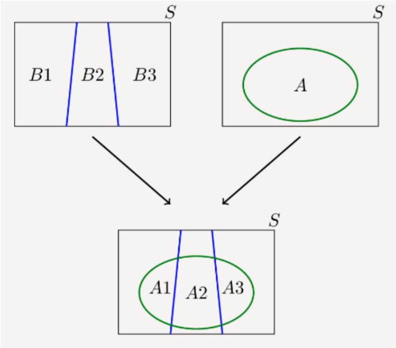
$$\begin{aligned} A &= \text{The first toss is heads;} \\ B &= \text{The second toss is heads;} \\ E &= \text{The eventually biased coin is used} \end{aligned}$$

Then A and B are conditionally dependent given E.

Marginalization and Law of Total Probability

Law of Total Probability[5]**:** If B1, B2, B3, . . . is a partition of the sample space S, then for any event A we add the amount of probability of A that falls in each partition

$$P(A) = \sum_{B} P(A, B) = \sum_{B} P(A|B)P(B)$$



[5] www.probabilitycourse.com/chapter1/1_4_2_total_probability.php

Bayes' Rule

Bayes' Rule is used to determine the probability or update the probability of an event based on prior knowledge or evidence relating to that event.

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$$\text{Posterior} = \frac{\text{Likelihood} * \text{Prior}}{\text{Evidence}}$$

Posterior: updated probability after the evidence is considered
Likelihood: probability of the evidence, given the belief is true
Prior: probability before the evidence is considered
Evidence: probability of the evidence

Bayes' Rule

$$P(Hypothesis|Evidence) = \frac{P(Evidence|Hypothesis)P(Hypothesis)}{P(Evidence)}$$

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Bayes' Example

Suppose you have tested positive for a disease. What is the probability you actually have the disease?

This depends on the prior probability of the disease:

- $P(T = 1|D = 1) = 0.95$ (likelihood)
- $P(T = 1|D = 0) = 0.10$ (likelihood)
- $P(D = 1) = 0.1$ (prior)

So $P(D = 1|T = 1) = ?$

Bayes' Example

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Prior: probability before the evidence is considered
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$$P(D = 1|T = 1) = \frac{P(T = 1|D = 1)P(D = 1)}{P(T = 1)}$$

Bayes' Example

(Solution: Use Bayes' Rule:

$$\begin{aligned} P(T=1) &= P(T=1|D=1)P(D=1) + P(T=1|D=0)P(D=0) \\ &= 0.95 * 0.1 + 0.1 * 0.90 = 0.185 \end{aligned}$$

$$P(D=1|T=1) = \frac{P(T=1|D=1)P(D=1)}{P(T=1)} = \frac{0.95 * 0.1}{P(T=1)} = 0.51$$

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Random Variable

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For example, let's flip a coin 10 times. $X(\omega)$ counts the number of Heads we observe in our sequence. If $\omega = HHTHTHHTHT$ then $X(\omega) = 6$. We often shorten this and refer to the random variable X.

Expectations

From our example, we see that X does not have a fixed value, but rather a distribution of values it can take. It is natural to ask questions about this distribution, such as "What is the average number of heads in 10 coin tosses?"

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This average value is called the **expectation** and denoted as $E[X]$. It is defined as

$$E[x] = \sum_{a \in \mathcal{A}} P[X = a] \times a$$

where $\mathcal{A}$ represents the set of all possible values $X(w)$ can take.

Expectation Practice

What is the expected value of a fair die?

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(Solution: X = value of roll

$$E[X] = \sum_{a \in \{1,2,3,4,5,6\}} \frac{1}{6} a$$
$$= \frac{1}{6} \sum_{a=1}^{6} a$$
$$= \frac{21}{6} = \frac{7}{2}$$

)

Linearity of Expectations

There are two powerful properties regarding expectations.

1. $E[X+Y] = E[X] + E[Y]$.
   This holds even if the random variables are dependent.
2. $E[cX] = cE[X]$, where c is a constant.

Note we cannot say anything in general about $E[XY]$.

Linearity of Expectation Practice

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X_1 = value of roll 1
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(Solution:

$$E[X_1 + X_2] = E[X_1] + E[X_2] = \frac{7}{2} + \frac{7}{2} = 7$$

)

Linearity of Expectation Practice 2

Suppose there are n students in class, and they each complete an assignment. We hand back assignments randomly. What is the expected number of students that receive the correct assignment? When $n = 3$? In general?
X = Number of students that get their assignment back
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(Solution:

$$\begin{aligned} E[X] &= E[X_1 + X_2 + \ldots + X_n] \\ &= E[X_1] + E[X_2] + \ldots + E[X_n] \\ &= \frac{1}{n} \times n = 1 \end{aligned}$$

)

Variances

Knowing the expectation can only tell us so much. We have another quantity used to describe how far off we are from the expected value. It is defined as follows for a random variable X with $E[X] = \mu$:

$$\mathrm{Var}[x] = E[(X - \mu)^2]$$

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The variance can be simplified as:

$$\begin{aligned} E[(X - \mu)^2] &= E[X^2 - 2\mu X + \mu^2] \\ &= E[X^2] - E[2\mu X] + E[\mu^2] \\ &= E[X^2] - 2\mu E[X] + E[\mu^2] \\ &= E[X^2] - \mu^2 \end{aligned}$$

Variance Properties

Constants get squared:

$$\mathrm{Var}[cX] = c^2 \,\mathrm{Var}[X]$$

For independent random variables X and Y, we have

$$E[XY] = E[X]E[Y]$$

and

$$\mathrm{Var}[X + Y] = \mathrm{Var}[X] + \mathrm{Var}[Y]$$

Variance Practice

Consider a particle that starts at position 0. At each time step, the particle moves one step to the left or one step to the right with equal probability. What is the variance of the particle at time step n?
$X = X_1 + X_2 + \ldots + X_n$

Variance Practice

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(Solution: Each X_i is 1 or -1 with equal probability.

$$\mathrm{Var}(X_i) = 1$$
$$\mathrm{Var}(X) = \sum \mathrm{Var}(X_i) = n$$

The expected squared distance from 0 is n.)

Discrete and Continuous Random Variables

Discrete Random Variables

- Takes countably many values, e.g., number of heads
- Distribution defined by probability mass function (PMF)
- Marginalization: $p(x) = \sum_y p(x, y)$

Continuous Random Variables

- Takes uncountably many values, e.g., time to complete task
- Distribution defined by probability density function (PDF)
- Marginalization: $p(x) = \int_y p(x, y) dy$

Probability Distribution Statistics

Expectation: First Moment, μ

$$E[x] = \sum_{i=1}^{\infty} x_i p(x_i) \qquad \text{(univariate discrete r.v.)}$$

$$E[x] = \int_{-\infty}^{\infty} x p(x) dx \qquad \text{(univariate continuous r.v.)}$$

Variance: Second (central) Moment, σ^2

$$\begin{aligned} \text{Var}[x] &= \int_{-\infty}^{\infty} (x-\mu)^2 p(x) dx \\ &= E[(x-\mu)^2] \\ &= E[x^2] - E[x]^2 \end{aligned}$$

I.I.D.

Random variables are said to be **independent and identically distributed** (i.i.d.) if they are sampled from the same probability distribution and are mutually independent.
This is a common assumption for observations. For example, coin flips are assumed to be i.i.d.

Questions?

Colab Notebook Group Activity

- You will be put into breakout rooms
- Look over the colab file as a group and discuss
- Discuss the Bayes' Rule as a group
- Complete the open-ended exercise on Bayes' Rule located at the end of the notebook

https://colab.research.google.com/drive/1_LFQD1YxWOW4JDuxneYMA03ajblGdZOB?usp=sharing